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Epoch of Matter-Radiation Equality:

After primordial synthesis of light elements, the universe consists of ionized Hydrogen and Helium (namely, protons and α particles), electrons, photons, neutrinos and dark matter. The energy density in the relativistic species is:

$$\rho_r = \rho_\gamma + \rho_\nu = \frac{\pi^2}{30} \times 2 \times T^4 + \frac{\pi^2}{30} \times \frac{7}{8} \times 2 \times N_\nu^{eff} \times \left(\frac{4}{11}\right)^{\frac{4}{3}} T^4 \Rightarrow$$

$$\rho_r = \frac{\pi^2}{30} \times 2 \left[1 + \frac{7}{8} \times \left(\frac{4}{11}\right)^{\frac{4}{3}} N_\nu^{eff}\right] T^4 \qquad (I)$$

In the standard model $N_\nu^{eff} = 3.04$. The energy density in the non-relativistic species is:

$\rho_m = \rho_B + \rho_{DM}$ (electron contribution to the energy density is negligible)

$$\rho_B \approx n_B m_p \quad , \quad \frac{n_B}{n_\gamma} \simeq 6 \times 10^{-10} \Rightarrow \rho_B \sim 6 \times 10^{-10} n_\gamma m_p \quad (m_p \sim 1 \text{ GeV})$$

$$n_\gamma = \frac{\zeta(3)}{\pi^2} \times 2 \times T^3 \Rightarrow \rho_B \sim \frac{\zeta(3)}{\pi^2} \times 1.2 \times 10^{-9} T^3 m_p$$

From the CMB data (most recently Planck Satellite) we know:

$$\rho_{DM} \sim 5.5 \rho_B \Rightarrow \rho_m \sim \frac{\zeta(3)}{\pi^2} \times 1.2 \times 5.5 \times 10^{-9} m_p T^3 \qquad (II)$$

At the epoch of matter-radiation equality, we have $\rho_r \sim \rho_m$.

This occurs when:

$$\rho_m \sim \rho_r \Rightarrow T_{eq} \sim O(eV) \qquad (III)$$

Since the universe is radiation-dominated until this moment, we can use $t \propto \frac{M_P}{T^2}$ and the fact that $t \sim 1$ sec when $T \sim O(MeV)$.

Taking heating of photons by electron-positron annihilation, we then find:

$$t_{eq} \sim 60{,}000 \text{ yr}$$

A more precise calculation yields $t_{eq} \simeq 50{,}000$ yr. Considering the current temperature of the CMB photons $T_0 \simeq 2.72 K \sim 2 \times 10^{-4}$ eV, we find the redshift at the matter-radiation equality to be:

$$z_{eq} \sim 4500 \qquad (IV)$$

From Eq. (III) it is seen that T_{eq} is much smaller than the ionization energy of the Hydrogen atom 13.6 eV. It may therefore seem that electrons were bound to form neutral Hydrogen atoms at this epoch. However, this is not the case due to the very small value of $\frac{n_B}{n_\gamma} \simeq 6\times10^{-10}$. This implies that the small number of energetic photons in the Wien tail of the blackbody distribution is enough to keep Hydrogen ionized at T_{eq}. As we will see later, formation of atomic Hydrogen happens later at the epoch of recombination when $T_{rec} \sim 0.3\,eV$ (corresponding to $t_{rec} \sim 400{,}000$ yr and $z_{rec} \sim 1100$).

Bounds on the Neutrino Mass:

The atmospheric neutrino oscillations observed in 1998 have confirmed non-zero mass for (some of) the neutrinos. These oscillations, however, give information about the mass difference of the neutrinos. Together with solar neutrino oscillation experiments, there are two such mass differences:

$$\Delta m^2_{atm} \sim 5\times10^{-3}\ eV^2 \quad , \quad \Delta m^2_{sol} \sim 10^{-2}\,\Delta m^2_{atm}$$

Additional information from laboratory experiments and/or

Cosmology are required in order to identify the absolute masses of neutrinos. The current laboratory bounds on the absolute neutrino mass are much weaker than the mass scales set by Δm_{atm} and Δm_{sol}:

$m_{\nu_e} \lesssim 1$ eV , $m_{\nu_\mu} \lesssim 150$ keV , $m_{\nu_\tau} \lesssim 18$ MeV

One can find much tighter bounds from cosmology. In the context of the standard model of cosmology, which is in very good agreement with various observations, the total energy density in the neutrinos cannot exceed that of the dark matter at the present time. Even very light neutrinos $m_\nu \sim 10^{-3}$ eV are in the non relativistic regime today ($T_0 \sim 2\times10^{-4}$ eV). Therefore:

$\rho_\nu = n_\nu \sum_\nu m_\nu$, $n_\nu = \frac{\zeta(3)}{\pi^2} \times \frac{3}{4} \times 2 \times \frac{4}{11} T^3$

$\rho_{DM} \sim 5.5\rho_B \Rightarrow \rho_{DM} \sim 5.5\, n_B m_p \sim 5.5\times6\times10^{-10}\, m_p n_\gamma$

$n_\gamma \simeq \frac{\zeta(3)}{\pi^2} \times 2 \times T^3$

Therefore the condition $\rho_\nu \lesssim \rho_{DM}$ will result in:

$$\sum_{N} m_N \lesssim 10 \text{ eV} \qquad (\mathrm{V})$$

This is the so-called Cowsik-McClelland bound on light neutrino masses. It is very impressive as compared to the laboratory bounds particularly for ν_μ and ν_τ.

However, within the standard model of cosmology, this is a conservative bound. It is derived assuming that light neutrinos can constiute up to 100% of dark matter in the universe. This, however, leads to the "hot dark matter" scenario that is now ruled out by observations. This implies that light neutrinos can be only a small fraction of the total dark matter. In fact, much tighter bounds on $\sum_N m_N$ can be obtained from cosmological observations. For enamaple, the CMB data set a very stringent bound:

$$\sum_{N} m_N \lesssim 0.22 - 0.66 \text{ eV} \qquad (\mathrm{VI})$$

The range depends on which sets are data used (0.22 eV limit includes BAO "Baryon Acoustic Oscillations" data as well as CMB data).